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Original Article

Study of Recent Advances in Thermoelasticity

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Abstract

This review provides an extensive examination of recent advancements in the domain of thermoelasticity, with a particular focus on generalized theories that account for the second sound effect. Contemporary investigations have increasingly centered on the refinement and extension of generalized thermoelastic models. In contrast to classical thermoelastic theory, which is based on parabolic heat conduction, these generalized approaches typically incorporate hyperbolic-type heat conduction equations, motivated by experimental observations that reveal wave-like thermal behavior. Various researchers across multiple disciplines have contributed to the development and application of these models, addressing a wide spectrum of physical problems and elucidating the unique characteristics inherent to each formulation. The review systematically explores the theoretical underpinnings, mathematical frameworks, limitations, and analytical or numerical methods employed to solve problems involving different geometrical configurations and boundary conditions. Although classical thermoelasticity remains a valuable tool for evaluating temperature distributions, thermal stresses, and mechanical responses, generalized theories offer enhanced accuracy in modeling non-Fourier heat conduction phenomena.

Furthermore, a comparative analysis of solution strategies indicates that numerical approaches—such as the generalized differential quadrature method and finite element analysis—exhibit greater efficacy in addressing strongly nonlinear and geometrically complex problems. Conversely, analytical techniques, particularly those utilizing Laplace and Fourier transforms, are more suitably applied to problems involving memory effects or fractional-order derivatives. This review delineates the underlying theoretical constructs, mathematical models, inherent limitations, and solution methodologies employed for various geometrical domains and loading scenarios. It emphasizes that, although classical thermoelasticity provides a foundational framework, it lacks the capability to accurately capture non-Fourier heat conduction behavior observed in contemporary engineering applications. As a result, the adoption of generalized thermoelastic models becomes essential for achieving realistic and reliable thermal analyses.

Keywords: *Thermoelasticity, Temperature, Thermal stresses, thermoelastic medium.*

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INTRODUCTION:

Thermoelasticity theory represents an interdisciplinary framework that combines the fundamentals of elasticity and heat conduction, emphasizing the mutual interaction between thermal and mechanical fields. Specifically, it examines how temperature variations influence the deformation of elastic solids and how mechanical deformation, in turn, affects heat transfer. Thermal stresses emerge when the rate of change in internal heat generation or thermal boundary conditions is not synchronized with the structural response, necessitating the solution of coupled thermoelastic equations to determine the distributions of temperature and stress. In the classical formulation, the heat conduction equation is parabolic in nature, which inherently assumes an unphysical infinite speed of thermal wave propagation. To overcome this limitation, several generalized theories have been developed in recent years, introducing modifications to the classical Fourier law by adopting hyperbolic heat conduction models that allow for finite thermal wave speeds and improved physical realism. Among these models, the Green–Naghdi theory offers three distinct types of constitutive relations, each describing different thermomechanical coupling behaviors. Ignaczak further advanced the field by formulating a unified framework that integrates features of both the Green–Lindsay and Lord–Shulman models. This review aims to provide a comprehensive synthesis of contemporary research in generalized thermoelasticity, with an emphasis on recent developments in modified theories and their practical applications.

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It systematically examines prominent models such as the Lord–Shulman, Green–Lindsay, Green–Naghdi, dual-phase-lag, and multi-phase-lag formulations, highlighting their relevance across various materials and geometrical configurations. Additionally, the specific geometries considered and the analytical or numerical methods employed in each study are discussed in detail.

GREEN–LINDSAY (GL) THEORY OF GENERALIZED THERMOELASTICITY

Green and Lindsay [1] initially introduced a model known as the GL theory, which is classified as a temperature-rate-dependent thermoelasticity model. In this formulation, heat flux, entropy, and stress are influenced by both temperature changes and their rates. The model enhances the generalized thermoelasticity framework by incorporating two distinct relaxation times and is based on a deviation from Fourier's classical heat conduction law, particularly for media with central symmetry.

Zarmehri et al. [2] applied the GL model to derive stress intensity factors for a stationary crack in isotropic two-dimensional domains subjected to thermal shocks. Aouadi, Ciarletta, and Tibullob [3] analyzed the temperature and displacement responses of a viscoelastic medium with a deformable base under the same theoretical framework. Shivay and Mukhopadhyay [4] used a modified GL model to investigate the behavior of a functionally graded hollow disk under thermal loading. They employed the Newmark integration scheme alongside the Galerkin finite element method to solve the problem in the time domain, noting significant discrepancies between the original and modified GL models.

Othman and Edeeb [5] studied the effect of rotation on the thermoelastic response of a medium with temperature-dependent elastic moduli and porosity, using the GL model. Singh, Gupta, and Mukhopadhyay [6] developed fundamental solutions for a modified GL theory that incorporates both temperature and strain rate dependencies. They applied Laplace transform techniques to derive solutions in the Laplace domain, followed by inverse transformation to obtain temperature and displacement responses in the physical domain.

LORD–SHULMAN (LS) THEORY OF GENERALIZED THERMOELASTICITY

The classical model is based on the Fourier's law [1] which is given as $q_i = -kT_i$, where T_i and q are temperature gradient and heat flux, respectively. LS model is a theory of thermoelasticity involving a hyperbolic heat transfer equation that resulted from substituting the Fourier's heat conduction law by Cattaneo–Vernotte law [5]:

$$q + \tau_1 \frac{\partial q}{\partial t} = -k\nabla t$$

Where τ_1 is the relaxation time of the heat expressed as the lag time required to construct the steady condition of the heat conduction as a gradient of temperature is considered.

The final related heat transfer equation is expressed in Equation given by

$$k\nabla^2 T + \left(Q + \tau_1 \frac{\partial Q}{\partial t}\right) = \rho C\left(\frac{\partial T}{\partial t} + \tau_1 \frac{\partial^2 T}{\partial t^2}\right)$$

Where ρ is density, C is specific heat capacity.

A considerable body of research has focused on both transient and steady-state phenomena within the domain of thermoelasticity. For time-dependent analyses, the Laplace transform technique is widely utilized due to its effectiveness in handling temporal variables. Chen and Dargush [7] proposed a boundary element method tailored for solving transient coupled thermoelastic equations. Rabizadeh et al. [8] developed an adaptive computational strategy incorporating goal-oriented error estimation to enhance the accuracy of solutions to coupled thermoelastic systems. Kiani and Eslami [9] analyzed a homogeneous, isotropic layer using the Lord–Shulman (LS) generalized thermoelastic model. To solve the resulting system of coupled partial differential equations, they employed the generalized differential quadrature (GDQ) method. Their results revealed that the differences between linear and nonlinear thermal responses become increasingly significant with higher values of relaxation time, thermoelastic coupling coefficients, and thermal shock intensity.

Entezari and Kouchakzadeh [10] derived an analytical solution using the finite Hankel transform to investigate the thermoelastic behavior of a rotating disk subjected to simultaneous mechanical and thermal shocks within the framework of the Lord–Shulman (LS) theory. Their findings revealed that rotational motion induces a temperature gradient as a result of the coupling between mechanical strain and thermal fields. Furthermore, it was observed that an increase in angular velocity amplifies the magnitude of temperature oscillations. In a related study, Mirzaei [11] employed the LS model in conjunction with the generalized differential quadrature (GDQ) method to analyze the thermal response of a one-dimensional bounded strip exposed to a thermal shock, providing a detailed characterization of the transient thermomechanical behavior.

GREEN–NAGHDI (GN) THEORY OF GENERALIZED THERMOELASTICITY

Green and Naghdi [12] presented three theories that were referred to as GN-I, GN-II, and GN-III theories. They proposed the heat conduction law of form three, where heat flux was considered as a mixture of models I and II. So, the Fourier's law is substituted by

$$q(x,t) = [K\nabla\theta(x,t) + K^*\nabla\nu(x,t)]$$

where K is the thermal conduction coefficient and K^* is the Green–Naghdi coefficient.

Hussein [13] derived an analytical solution for the coupled thermoelasticity equations in nanobeam and microbeam resonators using the Laplace transform method. The study employed the Green–Naghdi (GN) model in conjunction with the nonlocal Rayleigh beam theory to obtain closed-form expressions for lateral deflection and temperature distribution. The findings demonstrated that the proposed analytical approach is well-suited for analyzing such systems, with no restrictions on the geometry of the microbeam or nanobeam, the nature of the thermal shock, or the material properties involved [13].

To examine transient behavior in a fiber-reinforced anisotropic thick plate subjected to a thermal source, Sur and Kanoria [14] developed a new magneto-thermoelastic model based on the GN theory. They advanced the Laplace–Fourier double transform technique to solve the resulting dimensionless coupled equations.

Ezzat and Bary [15] proposed a generalized GN model incorporating fractional time-derivative heat conduction. They applied the Laplace transform to derive analytical expressions in the Laplace domain and employed inverse Laplace techniques, using Fourier series expansions, to obtain time-domain numerical results [15]. Their study revealed that the presence of a magnetic field enhances the propagation speed of dilatational elastic waves.

Kaur and Lata [16] investigated an axisymmetric problem in a two-dimensional transversely isotropic magneto-thermoelastic medium subjected to an inclined mechanical load, utilizing the GN theory of type III.

DUAL-PHASE-LAGS (DPL) THEORY OF GENERALIZED THERMOELASTICITY

A generalized thermoelasticity model with dual-phase lag was defined by Tzou (1995) [17], who presented two different phase lags in the heat conduction Fourier's law. The first one is for the heat flux vector, and also the second is for the gradient of temperature. Fourier's law is substituted as below using their proposed model:

$$q(x, t + \tau_q) = -K\nabla\theta(x, t + \tau_\theta)$$

Which expresses a procedure in which a gradient of the temperature considered through a material volume at the time of $t + \tau_\theta$ will not present an increase in a thermal flux at a point of x through that volume until the later time of $t + \tau_q$. Following that, Green and Naghdi (1993) [18] proposed a new theory of thermoelasticity model without considering energy dissipation (GN type II), where τ_θ and τ_q are dual-phase-lag terms.

Karmakar et al. [19] explored the thermoelastic behavior—encompassing temperature, displacement, strain, and stress—of an infinite isotropic elastic medium containing a spherical cavity, utilizing the Dual-Phase-Lag (DPL) theory. The solution was obtained through the application of the state-space method in conjunction with a numerical Laplace transform inversion technique. Their study demonstrated that, under the influence of high-intensity, ultra-short-duration thermal loads, heat conduction is characterized by the presence of two distinct temperature fields: the thermodynamic temperature and the conductive temperature.

In another investigation, Kumar et al. [20] employed the integral transform method (ITM) and its inverse to examine the thermoelastic response of a thick circular plate subjected to an axisymmetric heat source, which varied along both the axial and radial directions. Further, Khamis et al. [21] (2020) developed a coupled plasma-thermal model integrated with a modified multi-phase-lag generalized thermoelastic theory to analyze a semi-infinite elastic semiconductor medium. Their work provided valuable insights into the complex coupling effects among thermal, mechanical, and electromagnetic fields in such advanced material systems.

CONCLUSION

This review has provided a comprehensive overview of recent advancements in the field of thermoelasticity, with particular emphasis on generalized models that incorporate the second sound phenomenon. The models discussed include the Multi-Phase-Lag (MPL), Lord–Shulman (LS), Green–Lindsay (GL), and Dual-Phase-Lag (DPL) theories. Although the concept of second sound has been recognized for decades, notable progress in the formulation and application of these models has occurred in recent years. It has been observed that, in coupled thermomechanical wave propagation, the amplitude of the temperature field associated with the displacement field exhibits a slight increase with higher frequency.

Additionally, a comparative evaluation of contemporary studies reveals that numerical techniques offer enhanced performance when addressing strongly nonlinear problems, while analytical methods are more suitable for systems involving memory-dependent behavior and fractional or integral formulations. The structure of this review encompasses the development of theoretical frameworks, mathematical modeling approaches, inherent constraints, and solution strategies applied to diverse geometries and loading scenarios. Overall, the findings highlight the limitations of classical thermoelastic theory in capturing complex thermal responses, underscoring the necessity of employing generalized thermoelastic models for more accurate simulation of heat conduction and deformation in advanced engineering applications.

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Conflicts Of Interest

There are no conflicts of interest.

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