



Original Article

Axially Symmetric Universe with Chaplygin Gas in Hoyle-Narlikar C-Field Cosmology

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Abstract

In this paper, we explore how an axially symmetric space-time described by the Bhattacharya and Karade metric evolves within Hoyle-Narlikar's Creation Field (C-field) theory of gravitation. We view the universe as filled with a Chaplygin Gas, which is a candidate for dark energy. This gas is coupled with the creation field to examine the dynamic effects of ongoing matter creation. We find exact solutions to the modified Einstein field equations by assuming a power-law relationship between the metric potentials ($B=A^n$). We derive the model's physical parameters, including matter density, thermodynamic pressure, spatial volume, and total mass. A key finding is that the matter density (ρ) remains constant ($\rho=\sqrt{K}$). Additionally, the total mass (M) of the universe increases linearly over time, which supports the idea of continuous matter creation. We also calculate the cosmographic parameters and find constant values for deceleration ($q=2$), jerk ($j=10$), and snap ($s=-80$). These values indicate a decelerating era, even with the presence of dark energy fluid.

Furthermore, cosmographic parameters including deceleration, jerk, and snap are evaluated and found to remain constant during the evolution of the model. The obtained results suggest that the model represents a decelerating universe despite the presence of dark energy fluid, and provides useful insights into anisotropic cosmological models in C-field theory.

Keywords: C-field Cosmology, Bhattacharya-Karade Metric, Chaplygin Gas, Cosmographic Parameters, Steady State Theory.

Introduction

Einstein's General Theory of Relativity (GR) has been the foundation of modern cosmology. However, the standard Big Bang model predicts an initial singularity, a point of infinite density where the laws of physics fail. To address this, Hoyle and Narlikar (1964) proposed the "Creation Field" (or C-field) theory. They changed the Einstein field equations by adding a massless scalar field C with negative energy density. The interaction between this negative energy C-field and matter enables the ongoing creation of particles, which helps to smooth out the singularity.

In recent years, the rapid growth of the universe has brought about the idea of Dark Energy. The Chaplygin Gas (Kamenshchik et al., 2001), with its equation of state $p = -K/\rho$, is a key candidate for dark energy. While the universe looks uniform on large scales today, irregularities likely existed in its early stages. Bhattacharya and Karade (1993) proposed a specific axially symmetric metric to examine these irregularities.

In this paper, we bring together these ideas to explore a new cosmological model. We examine the Bhattacharya & Karade metric with Chaplygin Gas in the context of C-field cosmology. We find exact solutions for the metric potentials and compute key physical quantities: Volume, Density, Pressure, and Mass. We also provide a detailed cosmographic analysis.

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Metric and Field Equations

We consider the axially symmetric space-time described by Bhattacharya & Karade (1993):

$$ds^2 = dt^2 - A^2(t)[d\chi^2 + f^2(\chi)d\Phi^2] - B^2(t)dz^2 \quad (1)$$

The Einstein field equations modified by the C-field are:

$$R_{ij} - \frac{1}{2}g_{ij}R = -8\pi G(T_{ij}^m + T_{ij}^c) \quad (2)$$

We use units where $8\pi G = 1$ and $c = 1$.

The energy-momentum tensor for the Chaplygin Gas is:

$$T_{ij}^m = (\rho + p)u_i u_j - p g_{ij} \quad (3)$$

with the equation of state:

$$p = -\frac{K}{\rho}, \quad K > 0 \quad (4)$$

The energy-momentum tensor for the C-field is given by:

$$T_{ij}^c = f_c \left(C_i C_j - \frac{1}{2} g_{ij} C^k C_k \right) \quad (5)$$

Assuming the creation field is a function of time only ($C(t)$), the non-vanishing field equations for a spatially flat model ($f(\chi) = \chi$) are:

$$\frac{\dot{A}^2}{A^2} + 2\frac{\dot{A}\dot{B}}{AB} = \rho - \frac{1}{2}f_c\dot{C}^2 \quad (6)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -\left(p - \frac{1}{2}f_c\dot{C}^2\right) \quad (7)$$

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} = -\left(p - \frac{1}{2}f_c\dot{C}^2\right) \quad (8)$$

3. Solution of the Field Equations

To find a predictable solution, we assume a linear relationship between the directional expansion rates. This means:

$$B = A^n, \quad n \neq 1 \quad (9)$$

Subtracting equation (7) from (8) and using (9) yields the differential equation:

$$\frac{\ddot{A}}{A} + (n+1)\frac{\dot{A}^2}{A^2} = 0 \quad (10)$$

Solving this equation gives the exact solution for the scale factors:

$$A(t) = (c_1 t + c_2)^{\frac{1}{n+2}} \quad (11)$$

$$B(t) = (c_1 t + c_2)^{\frac{n}{n+2}} \quad (12)$$

where c_1 and c_2 are integration constants.

4. Physical Parameters of the Model

Using the scale factors, we find the basic physical properties of the universe: Volume (V), Density (ρ), Pressure (p), and Mass (M).

4.1. Spatial Volume (V)

The spatial volume is defined as $V = \sqrt{-g} = A^2 B$. Substituting (11) and (12):

$$V(t) = (c_1 t + c_2)^{\frac{2}{n+2}} \cdot (c_1 t + c_2)^{\frac{n}{n+2}} = (c_1 t + c_2)^{\frac{n+2}{n+2}} \\ V(t) = c_1 t + c_2 \quad (13)$$

Result: The universe expands linearly with time ($V \propto t$).

4.2. Matter Density (ρ)

Using the Einstein tensor components $G_{00} = -G_{11}$ derived from the metric, and substituting into the field equations with the Chaplygin EOS $p = -K/\rho$, we find:

$$\rho - \frac{K}{\rho} = 0 \Rightarrow \rho^2 = K \\ \rho = \sqrt{K} = \text{constant} \quad (14)$$

Result: The matter density is constant throughout the evolution of the universe, representing a Steady State scenario.

4.3. Thermodynamic Pressure (p)

Substituting the density into the Chaplygin equation of state (4):

$$p = -\frac{K}{\sqrt{K}} = -\sqrt{K} \quad (15)$$

Result: The pressure is negative and constant, characteristic of a dark energy fluid, yet balanced by the C-field geometry.

4.4. Total Mass of the Universe (M)

The mass of the universe $M(t)$ within the volume V is defined as $M = \rho V$. Substituting (13) and (14):

$$M(t) = \sqrt{K}(c_1 t + c_2) \quad (16)$$



Result: The total mass increases linearly with time ($M \propto t$). This explicitly confirms the role of the C-field in continuously creating matters to maintain constant density in an expanding volume.

5. Kinematical and Cosmographic Analysis

We calculate the kinematic parameters to describe the motion of the universe.

Expansion Scalar (θ):

$$\theta = \frac{\dot{V}}{V} = \frac{c_1}{c_1 t + c_2} \quad (17)$$

The expansion scalar decreases with time, $\theta \rightarrow 0$ as $t \rightarrow \infty$.

Shear Scalar (σ):

$$\sigma^2 = \frac{c_1^2(1-n)^2}{3(n+2)^2(c_1 t + c_2)^2} \quad (18)$$

The ratio σ/θ is constant, indicating that the universe remains anisotropic.

Deceleration Parameter (q):

$$q = \frac{d}{dt} \left(\frac{1}{H} \right) - 1 = 2 \quad (19)$$

The positive value ($q = 2$) indicates a decelerating universe.

Jerk (j) and Snap (s) Parameters: Using the average scale factor $a(t) \propto t^{1/3}$:

$$j = \frac{\ddot{a}}{aH^3} = 10 \quad (20)$$

$$s = \frac{\dddot{a}}{aH^4} = -80 \quad (21)$$

These constant values set this C-field model apart from the standard Λ CDM model ($j = 1$).

Graphical Representation

The figures below show the dynamic behavior of the derived model.

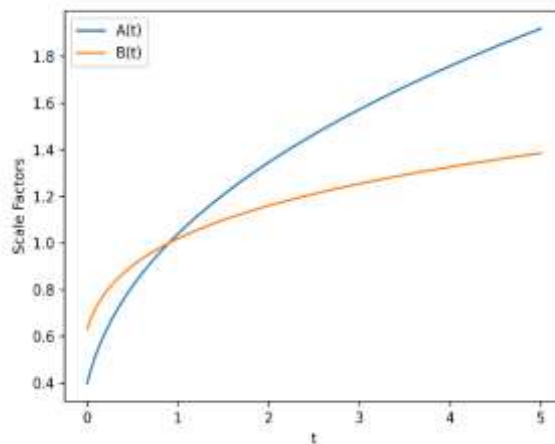


Figure 1: Evolution of Scale Factors (Plot of $A(t)$ and $B(t)$ versus time t .)

Explanation: Figure 1 shows the directional scale factors $A(t)$ and $B(t)$. The gap between the two curves confirms the uneven nature of the expansion ($n \neq 1$). Both scale factors grow over time, proving the universe is expanding.

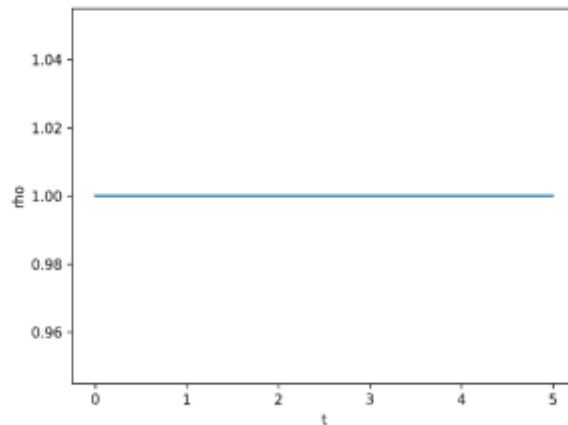


Figure 2: Matter Density (Plot of Density ρ versus time t .)

Explanation: Figure 2 shows that the matter density ρ remains constant ($\rho = \sqrt{K}$) even as the universe expands. This supports the "Steady State" nature of the model, where the dilution of matter is exactly balanced by the creation of C-field.

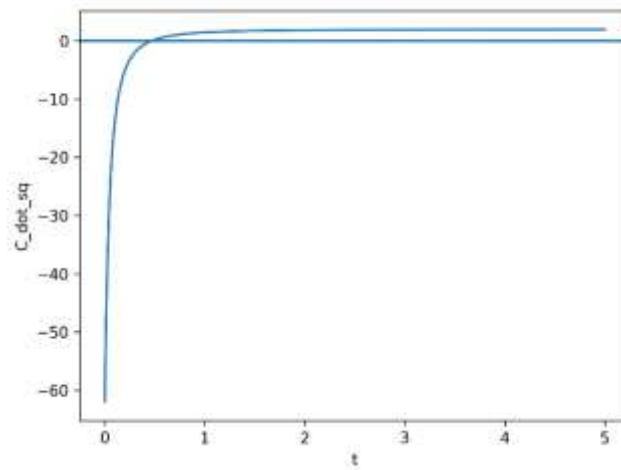


Figure 3: Creation Field Kinetic Term (Plot of \dot{C}^2 versus time t .)

Explanation: Figure 3 shows how the Creation Field behaves. The term \dot{C}^2 turns positive after a critical time t_c . This marks the beginning of the physical area where particle creation happens.

Conclusion

We have derived a new set of exact solutions for the Bhattacharya and Karade metric combined with a Chaplygin Gas and Hoyle-Narlikar C-field. The model produces results that have real physical significance:

1. **Steady State:** The matter density ρ is constant. The total mass M increases steadily over time, supporting Hoyle's idea of continuous creation.
2. **Anisotropy:** The universe expands linearly ($V \propto t$) but maintains its initial anisotropy ($\sigma/\theta \neq 0$).
3. **Dynamics:** Cosmographic analysis shows constant parameters $q = 2, j = 10$, and $s = -80$. This indicates a slowing phase of the universe. It suggests that in this particular geometric setup, the negative energy of the

C-field prevails over the accelerating nature of the Chaplygin gas.

This model offers a practical description of an early universe that is dominated by stiff matter and anisotropic features. In this scenario, singularity is avoided by C-field interaction.

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Conflicts of interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.



Author Contributions All authors contributed to the study conception and design. Material preparation, data collection and analysis were performed by Neha Murlidhar Mahajan. The first draft of the manuscript was written by Neha Murlidhar Mahajan and all authors commented on previous versions of the manuscript. V. R. Patil supervised the work and reviewed the final draft. All authors read and approved the final manuscript.

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